HOW COSTLY ARE THE PUBLIC SECTOR INEFFICIENCIES?

AN INTEGRATED THEORETICAL FRAMEWORK FOR ITS WELFARE ASSESSMENT

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Abstract

This paper provides a theoretical framework which integrates the conventional methodology for measuring the productive efficiency and the monetary assessment of social welfare changes associated with public sector performance. Two equivalent measures of social welfare changes generated by an improvement (or worsening) in productive efficiency are deduced using duality theory. The first one is obtained from the cost function, while the second one arises directly from the production function. Moreover, the paper induces the application of the theoretical framework proposed to empirical analysis.

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1 Introduction

Nowadays, an essential issue to be analysed in depth is the relationship between the productive efficiency of public sector and the potential budgetary savings associated with its improvement. Especially for advanced economies in which the current crisis effects are affecting the public finances in a more evident way. Quantifying these budgetary savings strongly constitute an alternative fiscal policy tool which goes beyond the traditional view of a fiscal consolidation (cut spendings or tax hikes). This measure is not only helpful for short-term consolidation but also it is required to guarantee a sound long-term growth path.

However, as commented in Pestieau and Tulkens (1993), it is not straightforward to properly measure the public sector performance. Beyond the recognition that evaluation of public sector policies requires the use of a multidimensional approach, they identify a number of difficulties. First, the objectives assigned to each production unit may not be always compatible with one another. Second, measuring the degree to which those objectives are satisfied force analytics to introduce some basic value judgements in order to weigh some partial indicators. Finally, the trade-off between allocative and non-allocative objectives may affect the management in these public sector units.

Initially, the literature on public sector focussed on several issues related to public choice theory (collective decision making, lack of competition and rent seeking), in order to understand the behaviour of public decision-makers (politicians, bureaucrats, lobbyists, etc.). Afterwards, since the late eighties the measurement of productive efficiency has received an increasing interest within the public economics area. This trend is even more evident for some specific sectors tipically provided by the public sector -health, education, etc.-. This growing literature has mainly focussed on developing quantitative methodologies (usually grouped into parametric and non-parametric methods) from which we may achieve empirical measures of (technical, allocative or overall) efficiency with which a number of units -assumed to be homogeneous- have produced the public good(s) and service(s). Thus, all these measures usually provide us one scenario to compare their performance.

1See Wilson (1989), Hettich and Winer (1993), Wolf (1993), Horn (1995) and Mueller (2003), among others, for further explanations about the topics dealt with from the point of view of this litetature -government size, budgetary incrementalism, X-iniciency, etc.-.

2See, among others, Afonso et al. (2005, 2010a, 2010b), Borge et al. (2008) and Casiraghi et al. (2009) for cross-country quantitative analyses on overall public sector performance and efficiency.

However, as Pestieau (2009) highlighted, it is not straightforward to move from theoretical concepts to empirical assessments, when dealing with issues related to the public sector efficiency. The difficulties come from both conceptual and feasibility perspectives. On the one hand, it has not been established so far how to break up the sources of allocative and technical efficiency as well as the existing relationship between them. For instance, a poor allocation of resources may disfavour to achieve high scores of technical efficiency. On the other hand, the non full availability of data -on inputs prices, for instance- may condition how we empirically quantify the public sector efficiency. In this respect, they compare both ideal and real-world assumptions to deal with this issue and finally determine which may be done under each scenario. Fundamentals of this latter literature are underlying in the construction of our theoretical proposal. Surely these contributions measuring the productivity of public services are very useful to improve the management of public resources. Nonetheless, we think there is lack of literature connecting these results with the potential budgetary gains that may arise from a reduction of public sector inefficiency.

In this vein, the OECD (2011) has recently highlighted the transcendence of implementing reforms addressed to increase the efficiency of public spending, specially for governments that are currently facing outstanding budgetary imbalances. In particular, the OECD refers to the need to improve the productivity of the public spending on education and health. In the first case, it is estimated that the gradual adoption of best practices in primary and secondary education could save resources around 0.5% of GDP (with country range from the 0.2 % to 1.2%), without compromising the current educational targets. In the case of health, the resources released by improvements in productive efficiency could be even higher, around 2% of GDP (range by country, between 0.4% and 4.8%).

Moreover, we think the monetary gains could be significant in terms of social welfare. In this respect, it is important to account not only budgetary savings but also the monetary gains in terms of income and wealth derived from consuming a better education and health. Furthermore, from the marginal cost of public funds perspective, we should also consider the reduction in deadweight losses caused by distortionary taxes which provide these resources released.

The aim of this paper is to provide a theoretical framework which allows consistently integrate the conventional methodology for measuring the productive efficiency and the monetary assessment of social welfare changes related to the public sector performance. Furthermore, we aim to discuss our theoretical proposal with a view to future empirical implementations. In particular, we deduce two measures of social welfare changes generated by an improvement (or worsening) in productive efficiency associated with the procurement of a public good. The first measure is...
obtained from the cost function, or in other words, from the supply side, while the second one arises directly from the production function. According to duality theory, both measures are equivalent and deducted from the same set of information.

The rest of the paper is organized as follows. In the second section, we introduce our theoretical framework, upon the basis of the conventional measures of productive efficiency (Farrell’s radial approach). In the third section, we present our integrated approach which combines different dimensions typically involved in policy-makers decisions (welfare changes, measures of inefficiencies, etc.). The fourth section discusses the most relevant issues in translating our proposal to potential empirical implementations. Finally, the last section concludes.

2 Theoretical framework

2.1 Recent concerns on Public Sector Efficiency (PSE)

The monitoring of public sector activity and the potential derivation of measures of the Public Sector Efficiency (PSE) clearly justify the increasing interest observed on analyses related to the Public Sector Performance (PSP, hereinafter). This section briefly discusses the recent evolution of literature focussed on the relevant concept, the Public Sector Efficiency (PSE, hereinafter), which refers to the efficient allocation and production of the public good and services. The existing literature comprise alternative approaches to measure -and evaluate- the PSP and, consequently, the PSE. A non exhaustive description of how this literature has evolved is next. Firstly, a growing number of studies (Afonso et al., 2005 and 2010b, Borge et al. 2008, and Clements, 2002, among others) translated the traditional approach used to analyse the productive efficiency of firms to the case of public sector units (countries, municipalities, schools, hospitals, etc.) with the aim of obtaining empirical measures of the PSE for a set of units and rank them. Secondly, some studies (Borge et al. 2008, among others) have also explored the identification of determinants of these empirical measures. An alternative perspective is considered by other authors (see Afonso et al., 2010a, Casiraghi et al., 2009, among others) in order to include the distributional concerns traditionally linked to the public sector activity into the efficiency analysis.

All in all, it can be observed that some caveats are still present. First, most of these analyses have focussed on the productive efficiency or technical efficiency ($\psi$). Thus, they have leaven out of the analysis issues related to the allocative efficiency ($\gamma$), a relevant component of the overall efficiency ($\eta$). This latter measure is our main interest in this paper. Second, the distributional concerns has not been yet fully incorporated to the analysis, although it is a component mostly
involved in policy-makers decisions.

Our paper aims to fulfill all these caveats by combining the elements presented; (i) empirical measures of efficiency, (ii) welfare impact and distributional concerns, (iii) a monetary valuation of inefficiencies measured.

2.2 The public sector

This section introduces the notation used in subsequent sections and models the Public Sector Performance according to a framework which could be adapted to very different analysis.

Our model can be briefly described as follows. The public sector produces a vector of goods and services \( Y = (y^1, ..., y^H) \) which we consider excludable unlike pure public goods\(^4\). Each \( y^h \) is produced by a public agency with the corresponding production function for the case of single output, such that,

\[
y^h = f_h(X) \tag{1}
\]

where \( X = (x_1, ..., x_n) \) is a vector of \( n \) inputs including fixed capital required for the activity and \( f_h \in S = \{(X,Y) : X \text{ can produce } Y\} \) with \( S \) the set of technologies.

The unitary price for each of these \( n \) inputs are included in the vector \( W = (w_1, ..., w_n) \). Consequently, the total cost of producing \( y^h \) \((c^h)\) is defined:

\[
c^h(y^h) = \sum x_i w_i \tag{2}
\]

Assuming \( H = 1 \), for the sake of clarity in the presentation, this theoretical framework allows us to introduce the notation used in posterior sections by defining formally all the standard concepts of efficiency -mentioned above- from the inputs-oriented perspective.\(^5\) First, departing from Farrell (1957) efficiency approach, given the minimum quantity of inputs needed for producing the level of output \( Y \) \((X^*)\), technical efficiency \((\psi)\) is defined as the ratio between \( X \) and \( X^* \), such that,

\[
\psi = \frac{\|X^*\|}{\|X\|} \tag{3}
\]

where \( \|\cdot\| \) represents the euclidean norm.\(^6\)

---

\(^4\)Rivalry and excludability are assumed to consistently reflect changes in the demand observed for each public good.

\(^5\)Analogous definitions can be found in the literature according to the output-oriented measures (see Coelli (2005) for a detailed comparison of both approaches). There are no divergences in the analyses carried out from both perspectives. Therefore, one of them can be excluded.

\(^6\)Alternative approaches to the concept of productive efficiency would imply to consider other definitions of norm. However, the subsequent formalization is valid for any alternative distance definition (and its respective norm).
Second, given the combination of inputs producing $Y$ at the minimum cost ($X^{**}$), the allocative efficiency ($\gamma$) is defined as the following ratio:

$$\gamma = \frac{\|X^{**}\|}{\|X\|}$$

(4)

Third, the overall efficiency can be defined as the product of expressions (3) and (4):

$$\eta = \frac{\|X^{**}\|}{\|X\|}$$

(5)

Finally, we derive the corresponding expression for $\eta$ in terms of production costs$^7$:

$$\eta = \frac{c^{**}}{c}$$

(6)

where $c$ and $c^{**}$ are, respectively, the actual level of production costs and the production costs corresponding to $X^{**}$, the efficient combination of inputs when producing $Y$, from the technical and the allocative perspective.

3 PSE analysis: an integrated approach

3.1 The "expenditure-efficiency" function

The framework described above can be observed from a different perspective, facing the dual version of the same problem. Under these circumstances, a given level of public output ($\hat{y}^h$) may be explained by the corresponding expenditure function ($c(\hat{y}^h)$), and the degree of overall efficiency ($\eta(\hat{y}^h)$). In other words, given the vector input prices ($W$), we can define an "expenditure-efficiency" function ($\phi$) which is implicit in the conventional production function with productive factors:

$$\hat{y}^h = f(X)|_W \rightarrow \hat{y}^h = \phi(c, \eta)|_W$$

(7)

First of all, from (6), we can express the budgetary cost of producing a quantity of public good ($\hat{y}^h$) from the vector of inputs ($X^{**}$) and the degree of overall efficiency reached in the productive process, $\eta$:

$$c(\hat{y}^h) = \eta^{-1} \sum_{i=1}^{n} x_i^{**} w_i$$

(8)

Second, by applying the inverse function theorem to the optimal technology $f_h^{**}$, the optimal quantities of each input ($x_i^{**}$) to produce $\hat{y}^h$ are obtained. Note that these values only depend on factor prices and technological parameters of the production function:

$$x_i^{**} = f_h^{**-1}(\hat{y}^h, W), i \in \{1, 2, ..., n\}$$

(9)

$^7$A proof of this equivalence between output and cost-oriented efficiency approaches can be seen in Coelli (2005).
Next, by combining (9) and (8), and solving for $\hat{y}^h$ we derive the expenditure-efficiency function $\phi$, as proposed:

$$
\hat{y}^h = \phi (c(\hat{y}^h), \eta)|_W
$$

(10)

To translate this general notation to our problem, the provision of this public good means to run into the public expenditure amount $c(\cdot)$, given the degree of efficiency with which the public agency produces ($\eta$).

3.2 Changes in the PSE, welfare impact and monetary valuation

This section presents an integrated approach which allows us to integrate the different dimensions involved in the evaluation of the Public Sector Performance; (i) changes in the degree of efficiency, (ii) welfare impacts linked to public policies, and (iii) monetary valuation of effects. The latter may facilitate the understanding of the inefficiency costs. Moreover, an improvement in the degree of efficiency will help to provide the same public good or service but with a lower level of spending.

For the sake of clarification, we detail our assumptions. First, in the following analysis it is assumed that any change in the degree of efficiency is exogenous. Nevertheless, as Gibbons (2005) discusses, the existence of internal disturbances in the organizations (miscoordination, lack of incentives, etc.) may be the source of inefficiencies. Second, the social welfare generated by consumption of public good ($y$) is measured in monetary value in the conventional way, that is, by computing the area under the curve of demand for the good and subtracting the cost of the inputs used in its production\(^8\). Additionally, to obtain accurate measurements of changes in consumer welfare we assume the demand functions involved to be compensated\(^9\). All in all, this theoretical framework contributes to measure welfare impacts linked to changes (improvements/worsening) in the degree of efficiency ($\eta$) with which the public good is produced.\(^10\) From Myrick-Freeman and Harrington (1990) analysis framework for private goods, next we introduce productive efficiency concerns on public production for evaluating its social welfare impact in monetary terms.

Thus, using our "expenditure-efficiency" function defined in (10), we can define the following social welfare function:

$$
\Omega = \Omega (X, W, \eta) = \int_0^y p(u)du - \sum_{i=1}^n x_i w_i
$$

(11)

\(^8\)Note that, as we did in the previous sections, hereinafter the notation is simplified to a single public good $y$ to highlight the underlying intuitions.

\(^9\)See Willig (1976) for a discussion on the accurate measurement of these areas.

\(^10\)Hereinafter, we consider a generic public good in order to simplify the notation. So, we avoid the superscript "h" used so far to refer to different public goods.
where \( p(\cdot) \) is the compensated demand function specified in its inverse form.

From (11) we obtain the first order conditions with respect to each inputs used \((x_i)\), such that,

\[
\frac{\partial \Omega}{\partial x_i} = p(y) \frac{\partial y}{\partial x_i} - w_i = 0, \quad i = 1, \ldots, n
\]

which determine the input demand functions \( x^{**}_i(w_i, \eta) \) for all \( i \). It should be noted here that these values are precisely those corresponding to the optimal vector of production factors, \( X^{**} \). It allows us to compute the optimal output level of public good for a given level of overall efficiency,

\[
y^{**}(\eta)|_W = \varphi(x^{**}_i(w_i, \eta), \eta)
\]

Likewise, we could define the social welfare function associated with the production of this public good by considering the overall efficiency \( (\eta) \) as main argument:

\[
\Omega(\eta)|_W = \varphi(x^{**}_i(w_i, \eta), \eta)
\]

Applying the envelope theorem to the algebraic analysis described above, we obtain the following proposition.

**Proposition 1** The net welfare gain is the value of the marginal contribution, in monetary terms, brought about by a reduction (or increase) of overall inefficiency in the production function, so that,

\[
\frac{\partial \Omega}{\partial \eta} = p(y^{**}) \frac{\partial y^{**}}{\partial \eta} - \sum_{i=1}^n w_i \frac{\partial x^{**}_i(\cdot, \eta)}{\partial \eta} = p(y^{**}) \varphi(\eta)(x^{**}_i(w_i, \eta), \eta)
\]

Some interesting implications are next. First, this result defines a relationship between the production function and the changes in welfare computed in the light of modification of the degree of efficiency. Second, it can be observed that, under full productivity of all inputs, the value generated by an infinitesimal improvement in productive efficiency is explained by the increase in the output generated. Third, from a different perspective, this gain could be seen as a closer approximation \((\varphi(\eta))\) to the optimal technology \((x^{**}_i)\).

Next, the dual version of this result is achieved. To do this, from (13) one can define the costs functions related to this production as a function of the optimal level of public good, the vector of inputs associated with the optimal technology and the degree of overall efficiency reached, so that,

\[
c(\eta) = c(y^{**}(\eta), \eta)
\]

Accordingly, considering the difference between consumer’s surplus and producer’s quasi-rents, we can rewrite (11) as,

\[
\Omega(y^{**}, \eta) = \int_0^{y^{**}} p(u) du - c(y^{**}, \eta)
\]
Note that $y^{**}$ guarantees that social welfare is maximized given that the price equals to the marginal cost of public good (equilibrium first order condition).\footnote{Hereinafter, for the sake of simplicity, we will not notate explicitly the dependence between the optimal level of output ($y^{**}$) and the degree of overall efficiency ($\eta$).}

\[ p(y^{**}) = \frac{\partial c(y^{**}, \eta)}{\partial y^{**}} \]  \hspace{1cm} (18)

Again, combining (17) and (18), the following proposition emerges.

**Proposition 2** The net welfare gain (loss) is the value of the marginal contribution, in monetary terms, brought about by the reduction (increase) of production cost as a consequence of an improvement (worsening) of the degree of overall inefficiency

\[ \frac{\partial \Omega(y^{**}, \eta)}{\partial \eta} = - \frac{\partial c(y^{**}, \eta)}{\partial \eta} \]  \hspace{1cm} (19)

**Proof.** Given (17), we compute the total derivative with respect to the degree of efficiency ($\eta$). That is,

\[ \frac{d \Omega(y^{**}, \eta)}{d \eta} = \frac{\partial \Omega(y^{**}, \eta)}{\partial y^{**}} \frac{dy^{**}}{d \eta} + \frac{\partial \Omega(y^{**}, \eta)}{\partial \eta} \]  \hspace{1cm} (20)

where:

\[ \frac{\partial \Omega(y^{**}, \eta)}{\partial y^{**}} = p(y^{**}) - \frac{\partial c(y^{**}, \eta)}{\partial y^{**}} \]  \hspace{1cm} (21)

and

\[ \frac{\partial \Omega(y^{**}, \eta)}{\partial \eta} = p(y^{**}) \frac{dy^{**}}{d \eta} - \left( \frac{\partial c(y^{**}, \eta)}{\partial y^{**}} \frac{dy^{**}}{d \eta} + \frac{\partial c(y^{**}, \eta)}{\partial \eta} \right) \]  \hspace{1cm} (22)

Firstly, as a consequence of (18), we could identify $\frac{d \Omega(y^{**}, \eta)}{d \eta}$ and $\frac{\partial \Omega(y^{**}, \eta)}{\partial \eta}$. Next, from (22), grouping conveniently and using again (18), we obtain the proposition. $\blacksquare$

**Corollary 1** An improvement in the degree of overall inefficiency always involves an increase in social welfare.

Again, some interesting conclusions can be derived. First, this result defines a relationship between the costs function and the changes in welfare computed when the degree of efficiency is modified. Second, these results can be understood as follows. The infinitesimal improvements in productive efficiency obtained lead to a reduction in the cost of production and, consequently, they are welfare enhancing. Third, combining propositions 1 and 2 we obtain that the two welfare measures proposed must coincide due to the duality in the relationship between the production function and the cost function, which is underlying in (7).
To conclude with this subsection, some interesting lessons could be extracted regarding the application of this approach to empirical analyses. First, the final results would lead to monetary valuations of the changes in the overall efficiency, which becomes a very interesting tool from the policy-makers perspective. Second, our approach integrates elements related to efficiency and others related to the equity, which allows to explore this classical trade-off (next subsection will explore this point in depth). Third, this approach requires an estimate of the production function and the cost function as well, which may limit its application when information on the production procedure and/or the production costs is limited.

3.3 Distributional issues

In this subsection, we analyse how the welfare gains from increased efficiency affect consumers of public goods and public sector itself as the producer. In this respect, we first identify the efficiency gains effects on consumer’s welfare. Let $\Omega^C$ be the measure of consumer surplus used (usually equivalent or compensatory variation), so that,

$$\Omega^C (\eta) = \int_0^{y^{**}(\eta)} p(u)du - p(y^{**}(\eta))y^{**}(\eta)$$

Then, the consumer’s marginal gain is,

$$\frac{\partial \Omega^C}{\partial \eta} = p(y^{**}(\eta))\frac{\partial y^{**}}{\partial \eta} - \frac{\partial p(y^{**})}{\partial y^{**}} \frac{\partial y^{**}}{\partial \eta} y^{**}(\eta) - p(y^{**}(\eta))\frac{\partial y^{**}}{\partial \eta}$$

and simplifying,

$$\frac{\partial \Omega^C}{\partial \eta} = -\frac{\partial p(y^{**})}{\partial y^{**}} \frac{\partial y^{**}}{\partial \eta} y^{**}(\eta)$$

Now, from the producer’s perspective, we repeat a similar strategy. First, we define the producer’s surplus in terms of $\eta$:

$$\Omega^S (\eta) = p(y^{**}(\eta))y^{**}(\eta) - c (y^{**}, \eta) \sum_{i=1}^{n} x_i^{**}w_i$$

where $x_i^{**}$ is determined by the $n$ input demand functions, $x_i^{**} (w_i, \eta)$.

Again, the producer’s marginal gain can be obtained by differentiating the previous expression:

$$\frac{\partial \Omega^S}{\partial \eta} = \frac{\partial p(y^{**})}{\partial y^{**}} \frac{\partial y^{**}}{\partial \eta} y^{**}(\eta) + p(y^{**}(\eta))\frac{\partial y^{**}}{\partial \eta} - c (y^{**}, \eta)\frac{\partial y^{**}}{\partial \eta} - \frac{\partial c (y^{**}, \eta)}{\partial \eta}$$

Taking into account eq. (18), we find that
In the light of the previous expressions, the following proposition can be proved.

**Proposition 3** An improvement in the degree of overall inefficiency always lead to an increase in consumer’s welfare. By contrast, this welfare gain is not guaranteed in the case of producers of public goods.

**Proof.** On the one hand, for consumers, this proof can be reduced to check the signs of the expressions mentioned above. As \( \frac{\partial p(y^* \eta)}{\partial y^* \eta} \leq 0 \) and \( y(\eta) > 0 \), depending on the sign of \( \frac{\partial y^*(\eta)}{\partial \eta} \), the consumer’s net welfare gain will be positive or negative. The optimal vector of inputs (from the technological and the minimisation of costs’ perspective) is taken as given in (13). As a consequence, a reduction of inefficiency may, in principle, lead to a decreased level of output -in equilibrium-. To clarify this latter statement, we differentiate the first order conditions mentioned above -in (18)- to achieve the following expression:

\[
\frac{\partial p(y^*)}{\partial y^* \eta} \frac{\partial y^*(\eta)}{\partial \eta} = \frac{\partial^2 c(y^*, \eta) \frac{\partial y^*(\eta)}{\partial \eta}}{\partial y^* \eta^2} + \frac{\partial^2 c(y^*, \eta)}{\partial y^* \eta^2} \quad (29)
\]

Grouping conveniently:

\[
\frac{\partial y^*(\eta)}{\partial \eta} = \frac{\frac{\partial^2 c(y^*, \eta)}{\partial y^* \eta^2}}{\frac{\partial p(y^*)}{\partial y^* \eta} - \frac{\partial^2 c(y^*, \eta)}{\partial y^* \eta^2}}
\]

On the one hand, looking at the denominator, it is straightforward to establish that \( \frac{\partial p(y^*)}{\partial y^* \eta} - \frac{\partial^2 c(y^*, \eta)}{\partial y^* \eta^2} < 0 \). On the other hand, any improvement in \( \eta \) lead to reductions in costs. Thus, \( \frac{\partial^2 c(y^*, \eta)}{\partial y^* \eta^2} < 0 \) and, consequently, \( \frac{\partial y^*(\eta)}{\partial \eta} \) is always positive.

All in all, we have proved that consumer’s welfare increases can be derived from the response in the production costs to an improvement in overall efficiency.

On the other hand, for producers, using the price-elasticity of public good demand, defined as \( \varepsilon = \frac{p(y^*)}{y^*} \frac{\partial y^*(\eta)}{\partial \eta} \), which is negative by definition, we can prove that \( \frac{\partial \Omega^S}{\partial \eta} \) will only be negative if and only if \( \frac{\partial y^*(\eta)}{\partial \eta} > \varepsilon \frac{\frac{\partial c(y^*, \eta)}{\partial \eta}}{p} \).

That is, the difference between the social welfare change and the variation in the consumer surplus. ■
From proposition 3, the distribution of welfare gains derived from an improvement in the degree of efficiency may be established. Our results indicate that the determinants are the optimal output response to this increase and the price-elasticity of demand. In short, three different possibilities are achieved:

\[(i) \quad 0 < \frac{\partial y^{**}(\eta)}{\partial \eta} < \frac{\partial c(y^{**}, \eta)}{p} \quad \Leftrightarrow \quad \frac{\partial \Omega^C}{\partial \eta} > 0, \quad \frac{\partial \Omega^S}{\partial \eta} > 0 \quad (30)\]

\[(ii) \quad \frac{\partial c(y^{**}, \eta)}{p} < \frac{\partial y^{**}(\eta)}{\partial \eta} \quad \Leftrightarrow \quad \frac{\partial \Omega^C}{\partial \eta} > 0, \quad \frac{\partial \Omega^S}{\partial \eta} < 0 \quad (31)\]

### 3.4 Social Welfare changes over time

In order to show a different perspective of the conclusions described so far, we consider now an example to illustrate (and reinforce) the underlying intuitions. We consider a scenario in which the overall efficiency to produce the public good \( y \) improves between two moments in time, from \( \eta_0 \) to \( \eta_1 \). To quantify the value of social welfare generated by the change in the degree of efficiency, we may choose to integrate, alternatively, one of the two welfare change measures presented in Propositions 1 and 2, respectively, and use \([\eta_0, \eta_1]\) as integration interval:

\[
\Delta \Omega = \int_{\eta_0}^{\eta_1} p(y^{**}(\eta)) \varphi_\eta (x^{**}_i (w_i, \eta), \eta) \ d\eta = -\int_{\eta_0}^{\eta_1} c(y^{**}(\eta), \eta) d\eta 
\]

The direct quantification of \( \Delta \Omega \) from any of the two alternatives shown in (32) requires to determine the changes in the equilibrium output and in the optimal combination of inputs caused by the change in the degree of productive efficiency.

On the contrary, this computation may be simplified when information on production levels of public good before and after to the change analysed is available. To do this, using (11), we simply need to calculate the difference between initial and final social welfare values

\[
\Delta \Omega = \int_0^{y_1} p(u) du - c(y_1, \eta_1) - \int_0^{y_0} p(u) du + c(y_0, \eta_0) 
\]

By using this quantification, it can be observed how the potential welfare gains resulting from improved efficiency come from the displacement of the supply curve (as there is a reduction in the cost function). In other words, marginal cost of producing public good goes from \( \frac{\partial c(y, \eta_0)}{\partial y} \) to \( \frac{\partial c(y, \eta_1)}{\partial y} \).

Following to Myrick-Freeman and Harrington (1990), we can obtain an alternative expression for (33) by incorporating the change experienced by the cost function. To do this, we use the line integral of its gradient along any path between \((x_0, \eta_0)\) and \((x_1, \eta_1)\), and integrate along the line.
connecting them, such that\(^{12}\):

\[
\Delta \Omega = \int_{y_0}^{y_1} p(u) du - \int_{y_0}^{y_1} \frac{\partial c(y_0, \eta)}{\partial \eta} d\eta - \int_{y_0}^{y_1} \frac{\partial c(y_1, \eta)}{\partial \eta} dy
\]

(34)

Figure 1 shows the net social welfare gain expressed in (34) -green area-. For the sake of simplicity, we assume linearity for all the curves involved; both compensated public good demand, and marginal cost functions (pre and post).

According to the analysis presented above, we could additionally define welfare changes experienced by consumers and the public sector as public good supplier. On the one hand, consumers enhance their welfare by increasing the area under the compensated demand curve, as a consequence of the equilibrium price decrease, from \(p_0\) to \(p_1\).

Figure 2 shows the consumers’ welfare gain, which is represented by the total blue area. On the other hand, the net change in producer’s welfare results from compensating for the decrease in their initial surplus due to the lower resulting price -blue squared area-, with the new surplus caused by the reduction of costs charted in the new marginal cost function -red area-.

As a consequence, combining this graphical evidence with propositions presented above, we conclude that:

(i) For any \(\eta > 0\), \(\Delta \Omega = \Delta \Omega^C + (\Delta \Omega^S - \nabla \Omega^S) > 0\).

(ii) We have not any guarantee implying that \((\Delta \Omega^S - \nabla \Omega^S) > 0\).

4 Potential empirical implementation

One of the purposes of this paper is to provide an integrated theoretical framework which may support comprehensive empirical analyses measuring public sector efficiency. This section aims to discuss how the theoretical framework described in previous sections may be translated to future empirical applications. However, as Pestieau (2009) pointed out, it is not straightforward to move from theoretical concepts to empirical assessments, when dealing with issues related to the public sector efficiency.

The difficulties come from both conceptual and feasibility perspectives. On the one hand, it has not been established so far how to break up the sources of allocative and technical efficiency as well as the existing relationship between them. For instance, a poor allocation of resources may disfavour to achieve high scores of technical efficiency. On the other hand, the non full availability of data -on inputs prices, for instance- may condition how we empirically quantify

\(^{12}\)See Myrick-Freeman and Harrington (1990) for further details on the underlying method, which is out of the scope of this paper.
the public sector efficiency. In this respect, they compare both ideal and real-world assumptions to deal with this issue and finally determine which may be done under each scenario.

Interestingly for policy-oriented exercises, the potential empirical implementation of the proposal presented in this paper would lead to monetary valuations measuring the changes in the overall efficiency. Moreover, our framework integrates elements related to efficiency as well as those related to distribution (to what extent this paper revisits the trade-off efficiency-equity). We consider that the current state of the art could perfectly deal with these empirical challenges, both when measuring the performance of public units and estimating the functions involved. Particularly, the following information would be required: (i) a vector containing the unitary price for each input, (ii) an estimation of the demand function and, finally, (iii) an estimation of the production function (cost function) in the primal (dual) approach.

Logically, we are aware that implementing our proposal requires some "extra" information about prices. In this sense, some recent studies have wisely dealt with this caveat (i.e. Gronberg et al., 2012). Particulary, in this paper the authors estimate a translog stochastic cost frontier model using panel data for charter campuses and traditional public campuses in Texas over the five-year period 2005-2009. They model expenditures per pupil as a function of three output indicators \( q \), two measures of input prices \( w \) and five environmental factors \( x \).

Furthermore, another decision to be made is which efficiency measurement methodology is preferable according to the characteristics of the analysis. In this regard, DEA and stochastic methods have been extensively applied although we will focus here on the information requirements as a key element. On this particular matter, Gronberg et al. (2012) discusses how DEA may lead to major data requirements. All in all, this issue should be decided taking into account the own specific features of the public services provision analysed.

5 Concluding remarks

In the light of the current economic situation, the near future points to intense (supra-/intra-) national social debates on the monitoring of public sector performance (health, education, etc.). Particularly, advances economies are currently facing issues related to the reorganization of their welfare state. Within this framework, quantifying these budgetary savings strongly constitute an

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\(^{13}\)The description of specific alternatives to overcome this difficulty goes beyond the scope of this theoretical analysis.

\(^{14}\)For instance, when estimating cost functions, the DEA approach needs data on input prices, outputs, costs and inputs quantities whereas the latter one is not needed for stochastic methods.
alternative fiscal policy tool which goes beyond the traditional view of a fiscal consolidation (cut spendings or tax hikes). This measure is not only helpful for short-term consolidation but also it is required to guarantee a sound long-term growth path.

In this respect, important policy implications are derived from our results. First, this paper has presented an integrated approach which combines different dimensions involved in the usual policy-makers decisions (efficiency in the production of the public good, welfare impacts and monetary valuation). This proposal satisfies additional features in comparison to the usual methodologies extensively used so far. Mainly, our approach would allow to translate measures of (in)efficiencies into to a monetary value. Second, our proposal may be adapted to be used within a wide variety of empirical applications monitoring and/or evaluating the public sector performance. In this respect, we have identified the information requeriments. Finally, we have derived some analytical results which help to understand the underlying intuitions and their linkages.

Finally, this paper links and integrates two different fields growing in parallel so far. On the one hand, empirical analyses monitoring the public sector performance from the production side and, on the other hand, studies analysing the welfare implications of public policy-makers. For instance, this approach may provide guidance to the design of fiscal consolidation programs, so that they are compatible with a more efficient use of public resources.
References


Figure 1: Welfare impact brought by a PSE gain

Figure 2: Welfare impact brought by a PSE gain. Distributional issues